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MODELS OF CLOSED-LOOP USE/REPAIR CYCLES  
WITH APPLICATION TO THE DD-963 MAIN PRO-  
PULSION ENGINES

Samuel Stephenson Montgomery, et al

Naval Postgraduate School  
Monterey, California

September 1972

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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

MODELS OF CLOSED-LOOP USE/REPAIR CYCLES  
WITH APPLICATION TO  
THE DD-963 MAIN PROPULSION ENGINES

by

Samuel Stephenson Montgomery

and

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13. ABSTRACT <p>This thesis describes and compares two models which may be used as tools in making logistic support decisions for closed-loop systems, such as the use/repair cycle of the LM-2500 gas turbine engines. The models are then used to study the effect of changes in various system parameters. The models are a computer simulation developed by Cushing, Gautier, and Long [1] and an analytical model developed by Koenigsberg,[2].</p> <p>The steady-state behavior of both models is essentially the same, with the analytical model consistently more conservative than the simulation model.</p> <p>The analytical model has been adapted to a time-shared interactive computer system which allows the policy planner to immediately obtain effects on the system of changes in any one of seven parameters.</p>		

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Models of Closed-Loop Use/Repair Cycles  
With Application to  
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## I. INTRODUCTION

Litton Industries has been awarded a contract to build 30 SPRUANCE Class destroyers for the United States Navy with first delivery scheduled for December 1974. Each ship will be equipped with four General Electric LM-2500 gas turbine engines for main propulsion. The present maintenance plan provides for an age replacement policy, i.e., each engine will be removed upon attaining some prescribed number of operating hours or upon failure, whichever occurs first. Upon removal, the engine will be replaced by a ready-for-issue (RFI) engine obtained from an RFI pool and the removed engine sent to a central repair facility for repair and return to the RFI state, thus forming a closed-loop use/-repair system. The number of spare engines to buy, and the repair capability to establish, are to be determined.

This thesis describes and compares two models which may be used as tools in making logistic support decisions for closed-loop systems, such as the use/repair cycle of the LM-2500 gas turbine engines. The models are then used to study the effect of changes in various system parameters. The models are a computer simulation developed by Cushing, Gautier, and Long [1] and an analytical model developed by Koenigsberg [2].

The main decision variables are the pool size, number of repair lines, time to repair, and the ratio of time-between-overhaul (TBO) to annual hours per engine (TOR).

With TBO = 4,000 hours, TOR = 2,000 annual hours per engine, pool size = 19, repair time = 5 weeks, number of repair lines = 6, as the base case for analysis, the parameters were varied through ranges above and below their base case values. The results consistently reveal that the pool size can vary over a wide range, from a minimum of 12, with little or no effect on the status of the operating fleet. However, the status of the fleet is extremely sensitive to the other parameters, especially the number of repair lines, repair time, and TBO. For example, if all parameters except TBO are kept at base case values, and TBO is increased from 4,000 to 6,000 hours, the average number of engines short in the fleet drops from about 10 to virtually zero.

Both models give essentially the same steady-state results with the analytical model requiring much less computer time. This makes possible the use of the analytical model in a real time interactive computer system. However, the simulation model provides the flexibility to handle a larger range of problems such as those with transportation delays. Because of the long running time and other inherent characteristics of the simulation model, it cannot be used in a real time mode.

The analytical model has been adapted to a time-shared interactive computer system which allows the policy planner to immediately obtain effects on the system of changes in any one of seven parameters.

This thesis contains six sections, of which this is the first, and 2 Appendices. Section II contains a description of the system and a description of the simulation and analytic models. Section III contains a comparison of the two models under steady-state conditions. Section IV contains the analysis of the steady-state results obtained with the analytic model and effects of transportation delays and spare engine buying plans obtained with the simulation model. Section V contains the explanation of the interactive computer model. Section VI summarizes this study. Appendix A contains sample inputs and outputs of the interactive computer program. Appendix B contains the simulation and analytic model computer programs.

## II. THE MODELS

### A. SYSTEM TO BE MODELED

Before starting any discussion of the two models, we describe the system to be modeled. The SPRUANCE CLASS destroyers are to be powered by the General Electric LM-2500 gas turbine engines. The maintenance policy for these engines states that only a minimum amount of onboard maintenance is to be performed by ship's personnel. Instead, the engines are designed for easy removal and replacement, so that failed engines can be shipped to a central rework facility for maintenance. This sort of maintenance policy requires that spare engines be purchased to replace those that are undergoing rework. The purpose of this is to keep the fleet operational as much of the time as possible by replacing failed engines.

The typical cycle of an engine begins with an engine operating in the fleet. Based on a policy of age replacement, engines are removed from the fleet either when they have accumulated a specified number of operating hours, or when an actual failure occurs prior to the specified removal time. When an engine has been removed, it is transported to the rework facility. This shipping causes a delay in the engine's arrival at the rework facility. Once at the rework facility an engine may or may not begin rework depending on whether the rework facility is congested. However, once inducted into a rework line, each engine

requires a known amount of time to be reworked. When rework has been completed, the engine is placed in a ready-for-issue status to await a requirement for use in the fleet. There is usually no delay in transportation from the ready-for-issue pool to a ship, since a ship can predict in advance when an engine will reach its scheduled removal time. However, if a failure occurs early, the engine would experience a delay being sent to the ship. A block diagram of this system is shown in Figure 1.

#### B. THE SIMULATION MODEL

The computer simulation model is a General Purpose Simulation System/360 event stored routine. This model begins operation by generating a transaction, in this case the first ship delivery, which sets everything into motion. The effects of this transaction are as follows:

1. It causes the rotatable pool to be loaded in accordance with the loading instructions provided in the model. This model provides for loading the entire quantity at time zero or at any other time during the simulation, or to load increments up to some total quantity at various times during the simulation.

2. It causes the simulation clock to start. The model is run with increments of one week as the basic unit of time, i.e., the parameters are all scaled to weeks. The system is observed at the end of each week and the status of the operating fleet, rotatable pool, repair lines, etc. is recorded.

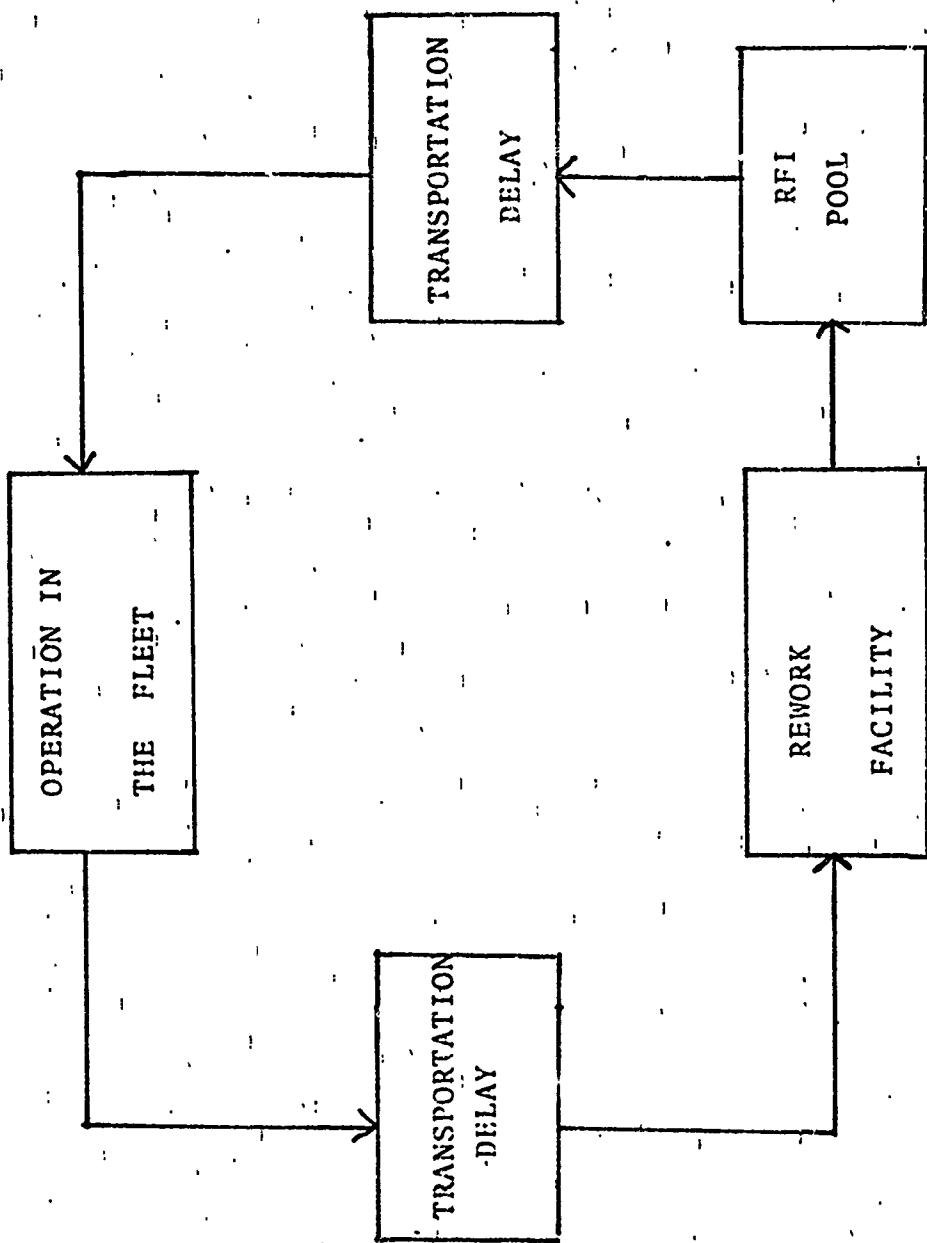


Figure 1. Block diagram of system to be modeled.

3. It causes operating units to be introduced. The model has the flexibility to provide for a build-up in the number of units operating by some prescribed delivery schedule, e.g., introduction of four engines with each DD-963 class destroyer in accordance with the ship's delivery schedule.

Various maintenance plans may be used, from the presently planned age-replacement policy, to one of removal based on some form of condition monitoring. In each case, a function determining the time to removal of each engine must be provided to the model.

A time to removal is assigned to each engine as it arrives in the operating fleet by a Monte Carlo technique, whether from new ship delivery or as a replacement engine drawn from the rotatable pool.

As the time period arrives for an engine to be removed, it leaves the fleet and a replacement is drawn from the pool, if available. Otherwise, replacement will be made with the first engine available at the pool. The removed engine proceeds to the repair facility, either directly, or through a transportation delay, if required.

Upon arriving at the repair facility, the engine is inducted on a first-come-first-served basis. If the repair lines are all busy, the engine is held in a queue until a repair line becomes available.

The engines proceed through repair in accordance with the repair function which assigns a time to repair for each engine inducted.

Upon completion of repair, the ready-for-issue engines then proceed directly to the rotatable pool where they remain until called for by the requirements of the operating fleet.

The parameters which must be specified as inputs to the simulation are:

1. The ship delivery schedule.
2. The number of engines per ship.
3. The function to generate engine removal times.
4. The rotatable pool loading plan, in time increments and quantity to be loaded each increment of time.
5. The transportation delays, if required.
6. The number of repair lines.
7. The function describing the time to repair.
8. The time for which the simulation is to run.
9. The standard GPSS statistics desired as output, or any special outputs desired.

The model typically requires about 2.5 minutes of computer time to provide statistics for ten iterations, each of 20 years duration for parameters in the ranges of interest.

### C. ANALYTICAL MODEL

The LM-2500 gas turbine engine use/rework cycle can also be described in terms of a finite cyclic queue model. We make the following three assumptions:

1. There is no delay in the transportation of engines to and from the rework facility.
2. The repair time at the rework facility is exponentially distributed with rate

$$\mu_1 = 1/T_1,$$

where  $T_1$  is the average time to rework an engine.

3. The time an engine operates in the fleet is exponentially distributed with rate

$$\mu_2 = 1/T_2,$$

where  $T_2$  is the mean number of weeks required for an engine to accumulate a specified number of operating hours.

The consequences of all three of the above assumptions will be discussed when this model and the simulation model are compared in section III. Under these assumptions, one can determine equations for the various system performance measures.

In order to discuss the model more easily, we define the following parameters:

$N$  = The total number of engines in the inventory. This includes both the number of engines to be installed in the fleet and the number of spare engines.

$A$  = The number of engines to be installed in the fleet.

$M$  = The number of repair lines in the rework facility.

Let us now define two stages for the system. An engine must be in one of these two stages. Stage 1 (The rework facility) is an  $M$ -server queue each with an exponential service time with mean rate  $\mu_1$ . When all  $M$  repair lines are busy an incoming engine waits in a queue for rework. Stage 2 (The fleet) is an  $A$ -server queue each with an exponential service rate  $\mu_2$ . When all ships have their full

complement of engines, a ready-for-issue (RFI) engine waits in a pool until there is a demand from the fleet for an engine.

The two stages form a closed system in that no engines enter or leave the system. Since N will remain constant, we can define

$P_i$  = Probability there are i engines in stage 1 (hence,  $N-i$  engines in stage 2).

For the finite cyclic queue just described, where the number of spare engines is greater than or equal to the number of repair lines in the rework facility, i.e.

$$N-A \leq M,$$

Koenigsberg in reference [2] determines the  $P_i$  to be:

$$P_i = (A^i/i!)(\mu_2/\mu_1)^i P_0, \quad 0 \leq i < M, \quad (1a)$$

$$= (A^i/M! M^{i-M}) (\mu_2/\mu_1)^i P_0, \quad M \leq i \leq N-A, \quad (1b)$$

$$= (A! A^{N-A} / (N-i)! M! M^{i-M}) (\mu_2/\mu_1)^i, \quad N-A < i \leq N. \quad (1c)$$

It is also necessary that

$$\sum_{i=0}^N P_i = 1. \quad (2)$$

These  $P_i$  will be used to calculate the various system measures.

Writing the ith equation as

$$P_i = a_i P_0, \quad (3)$$

then summing over  $i$  on both sides of (3) gives

$$P_0 \sum_{i=0}^N a_i = 1. \quad (4)$$

Therefore

$$P_0 = 1 / \sum_{i=0}^N a_i. \quad (5)$$

Substituting (5) into (3) we obtain

$$P_i = a_i / \sum_{j=0}^N a_j, \quad i = 0, 1, \dots, N. \quad (6)$$

Direct computation of the  $a_i$ 's leads to numerical problems. For example, let  $N = 140$ ,  $M = 6$ , and  $A = 120$ .

Then

$$a_{120} = \frac{120! (120)^{20}}{20! 6! 6^{114}}.$$

Direct computation of such a number in any machine will lead to problems since  $100!$  is of the order of  $10^{157}$ . Thus, we use the following recursive forms to calculate these coefficients:

Note that  $a_0 = 1$ , and let  $\rho = (\mu_2/\mu_1)$ .

Then

$$a_i = \frac{a_{i-1} A \rho}{i}, \quad 1 \leq i \leq M, \quad (7a)$$

$$= \frac{a_{i-1} A \rho}{M}, \quad M < i \leq N-A, \quad (7b)$$

$$= \frac{a_{i-1} \rho (N+2-i)}{M}, \quad N-A < i \leq N. \quad (7c)$$

In reference [2], Koenigsberg defines certain measures of system performance. The measures are interpreted in terms of the LM-2500, use/rework cycle.

$\bar{n}_1$  = Mean number of engines in the rework facility and adjoining waiting line,

$$= \sum_{i=0}^N i P_i.$$

$\bar{n}_2$  = Mean number of engines in the fleet and adjoining RFI pool,

$$= N - \bar{n}_1.$$

$\bar{w}_1$  = Mean number of engines awaiting rework,

$$= \sum_{i=M+1}^N (i-M) P_i.$$

$\bar{w}_2$  = Mean number of RFI engines,

$$= \sum_{i=0}^{N-A} (N-A-i) P_i.$$

$d_1$  = Percent time all repair lines are busy,

$$= (\sum_{i=M}^N P_i) \times 100$$

$H_1$  = Mean number of engines being repaired,

$$= \bar{n}_1 - \bar{w}_1.$$

$H_2$  = Mean number of engines operating in the fleet,  
=  $\bar{n}_2 - \bar{w}_2$ .

$U_m$  = Mean percent rework capacity used,  
=  $(H_1/M) \times 100$ .

$U_n$  = Mean percent of total engines in operation in  
the fleet,  
=  $(H_2/N) \times 100$ .

Three additional system measures not given by Koenigsberg in reference [2] are also included in the output. The first is cycle time (C). This is the average time required for an engine to complete a full cycle of the system. To derive this measure, we first note that the rate into a stage is equal to the rate out of the same stage. Then in steady state the rate out of the fleet stage is  $\mu_2 H_2$ . This is the rate everywhere in the system, so the cycle time (C) is given by

$$C = N/\mu_2 H_2.$$

To calculate the probability the fleet is short i or more engines ( $P_s(i)$ ), we use the following:

$$P_s(i) = 1 - \sum_{i=0}^{N-A+1} P_i.$$

To calculate the average number of engines short in the fleet ( $\bar{s}$ ), we use

$$\bar{s} = A - H_2 .$$

All of the above measures of system performance are printed in the output except  $\bar{n}_1$  and  $\bar{n}_2$  which are only used to calculate other measures.

There are seven parameters of the LM-2500 use/rework cycle which serve as the inputs to the model. They are:

1. Number of ships (S),
2. Number of engines per ship (E),
3. Number of spare engines (P),
4. Number of repair lines (M),
5. Average rework time per engine in weeks ( $T_1$ ),
6. Time between overhauls (TBO),
7. Engine operating hours per year (TOR).

The above seven parameters are converted internally in the program to the actual parameters used by the model in the following way:

$$A = S \times E,$$

$$N = A + P,$$

$$u_1 = 1/T_1,$$

$$T_2 = (TBO/TOR) \times 52 \times 0.9,$$

$$u_2 = 1/T_2$$

M is unchanged.

All of the above are self explanatory except  $T_2$ . This equation for  $T_2$  is explained fully in the section on comparison of the models. Briefly, it calculates a mean time to replacement consistant with the age replacement policy for engine removals.

### III. COMPARISON OF MODELS

The simulation model was developed because "... analysis of the DD-963 inventory problem disclosed that system complexities rendered analytical modeling infeasible." (See [1] , page 17) That is true if all of the transient problems are to be included. However, inclusion of the relatively short transient period in a long-term program could lead to decisions which are harmful in the long-term. For that reason, steady-state results from the simulation and the analytical model were compared to determine any differences in results from the two models. Since the analytical model has certain restrictions, e.g., no capability to handle transportation delays, the simulation model was modified to eliminate transportation delays and to eliminate the transient effects by introducing the 30 ships and loading the pool at time zero.

The two models were run with the same parameters over various ranges. The results of the two models were extremely close, with the analytical model consistently more conservative than the simulation model. Most important, as shown by Figures 2, 3, and 4, the response to changes in particular parameters followed the same pattern in the two models. As an example, the average number of engines awaiting repair plotted against rotatable pool size, with the 4,000 hour TBO, 2,000 annual hours per engine, repair time of 5 weeks, and

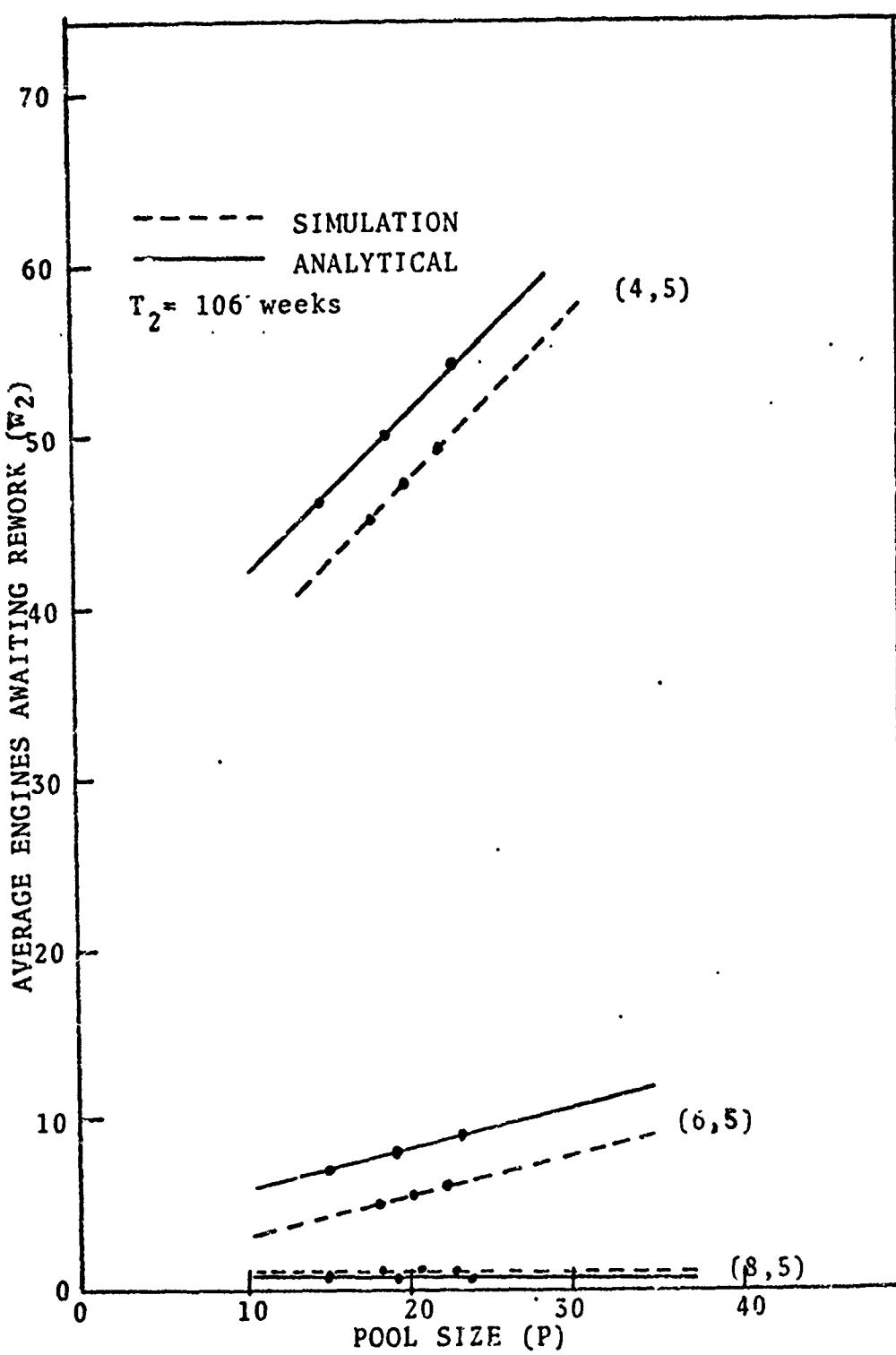


Figure 2.  $\bar{w}_2$  vs.  $P$ .

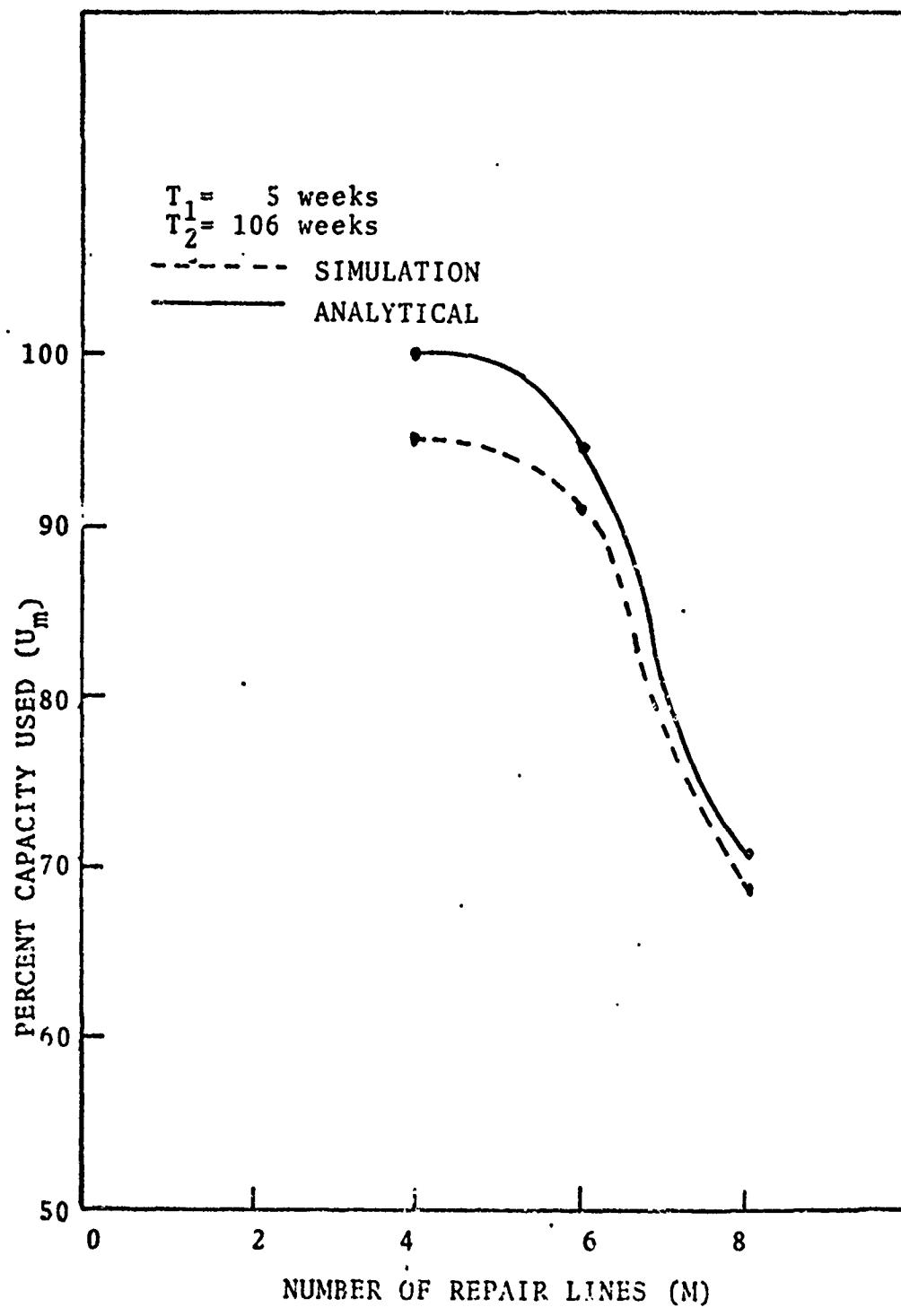


Figure 3.  $U_m$  vs. M.

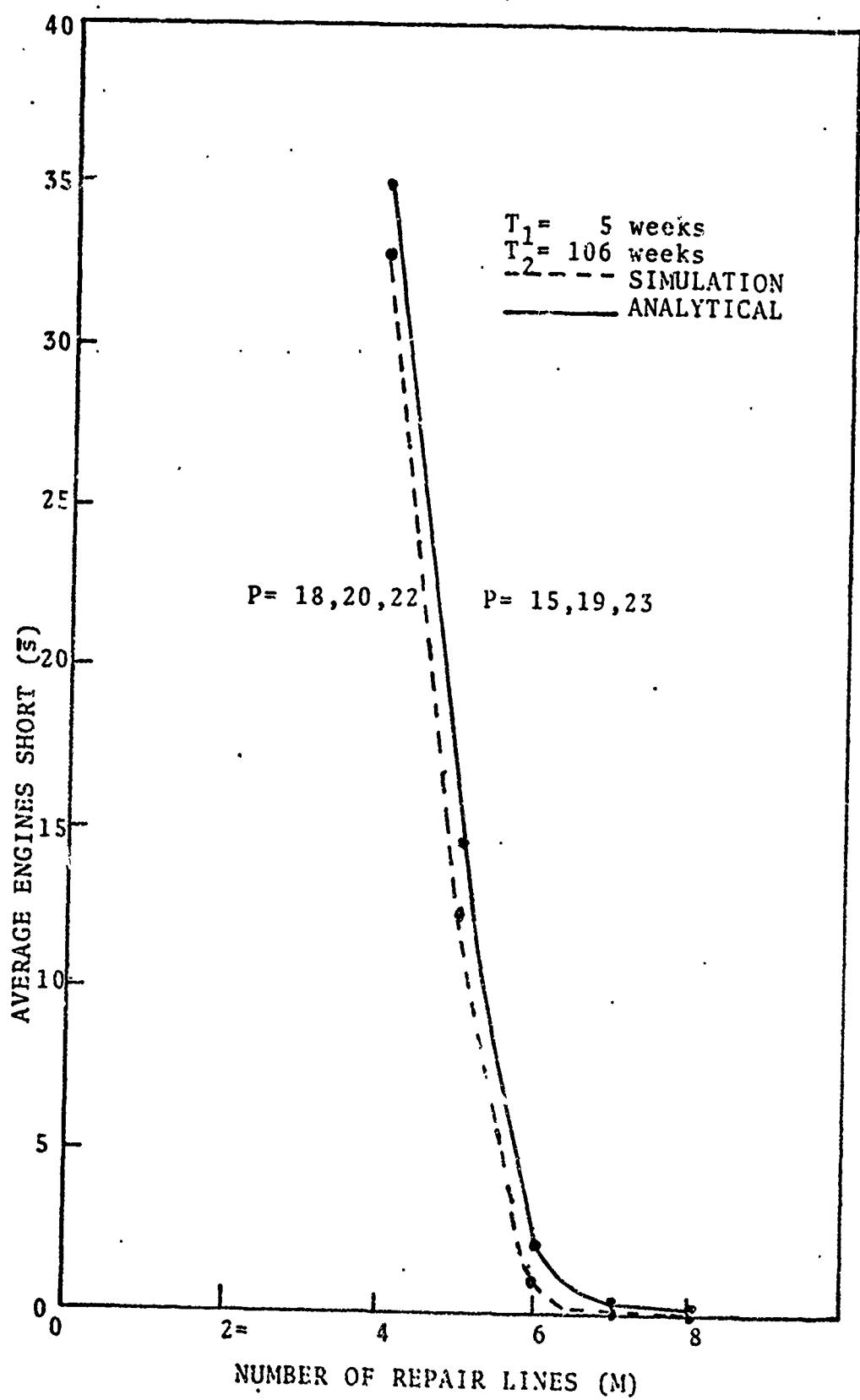


Figure 4.  $\bar{S}$  vs.  $M$

4, 6, and 8 repair lines, is displayed in Figure 2. While the number waiting is slightly less with the simulation model than with the analytical model, both models show the same behavior with a change in pool size. For a repair facility with only 4 repair lines and a pool size of 15, the waiting line is long, 42 for the simulation model and 46 for the analytical model. However, more important is that each additional engine added to the pool increases the size of the waiting line by one engine for both models. Similar results are evident with the other comparisons in Figures 3 and 4.

Although from the foregoing model comparison we conclude that both give the same steady-state results, it should be pointed out that the time in the fleet and repair time distributions used in the two models were very different. This shows that the analytical mode is quite robust and deviation from the exponential distributions do not lead to serious errors.

In the simulation model, the time an engine stays in the fleet has an empirically determined distribution function, constructed by Cushing, Gautier, and Long [1] using actual records of DD type operations. The analytical model has an exponential distribution function with a mean time in the fleet determined by the TBO and TOR. The repair time for the simulation is deterministic, whereas the analytical model uses an exponential distribution function

with a mean repair time equal to the deterministic value of the simulation.

In our comparison of the distributions of the time an engine stays in the fleet, the 4,000 hour TBO and 2,000 TOR operating profile are used. These result in a mean time in the fleet of 106 weeks which is used in the exponential distribution.

As previously explained in the simulation model, a time, in weeks, for an engine to reach 4,000 hours is a random variable which we will define to be  $X$ . The distribution of  $X$  is discrete with the following mass function:

x	P(X=x)
66 weeks	.009
72 "	.019
78 "	.019
84 "	.027
90 "	.046
96 "	.055
102 "	.076
108 "	.093
114 "	.120
120 "	.120
126 "	.120
132 "	.102
138 "	.074
144 "	.037
150 "	.029
156 "	.019
162 "	.039
168 "	.009
174 "	.009
elsewhere	.000

Once  $X$  is determined for an engine, there is a 0.8 probability that the engine will be removed at that time and a 0.2 probability that it will fail before the time  $X$ . The

distribution of the time for the early failure is assumed to be uniform between zero and X weeks.

The distribution of the time in the fleet is a random variable Y, a function of the random variable X, the U(0,X) random variable, and the probability of reaching the desired TBO. The conditional distribution of Y given X is:

$$P(Y \leq y | X=x) = \begin{cases} \frac{.2y}{x} & \text{for } y < x, \\ 1 & \text{for } y \geq x. \end{cases}$$

Figure 5 is a sample of the conditional distribution of Y.

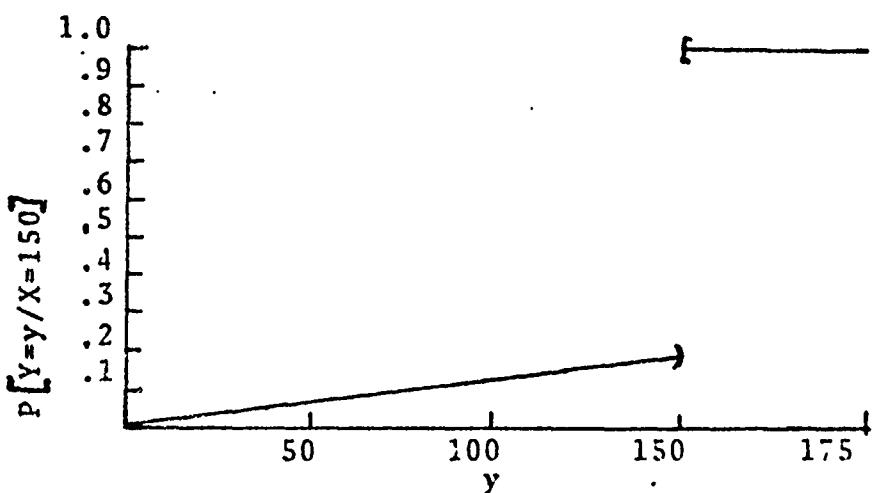


Figure 5. Sample Conditional Distribution of Y given X

The unconditional distribution of Y is

$$P(Y \leq y) = \sum_{x=0}^y p(x) + \sum_{x=y+1}^{174} \frac{.2y}{x} p(x).$$

A plot of this distribution function is displayed in Figure 6 along with the exponential distribution function with the same mean.

While the two functions are clearly quite different, the probability of a removal prior to 128 weeks of operation is approximately 0.74 for both distributions. There is a probability of approximately .16 that an engine stays in the fleet in excess of 174 weeks using the exponential distribution which is zero for the empirical distribution. This provision for times greater than 174 weeks prior to removal for some engines has a "real world" appeal.

This comparison was done with the 4,000 hour TBO and the 2,000 annual hours per engine TOR. The mean time in the fleet derived from the empirical distribution function for this combination is 106 weeks. This is derived by finding  $E[X]$  and then finding  $E[Y]$  as follows:

$$E[Y] = E[E[Y|X]] = E[\frac{.2X}{2} + .8X] = .9 E[X].$$

Cushing, Gautier, and Long [1] developed empirical distribution functions for various TBO and TOR combinations. Rather than calculate the mean of each of these functions

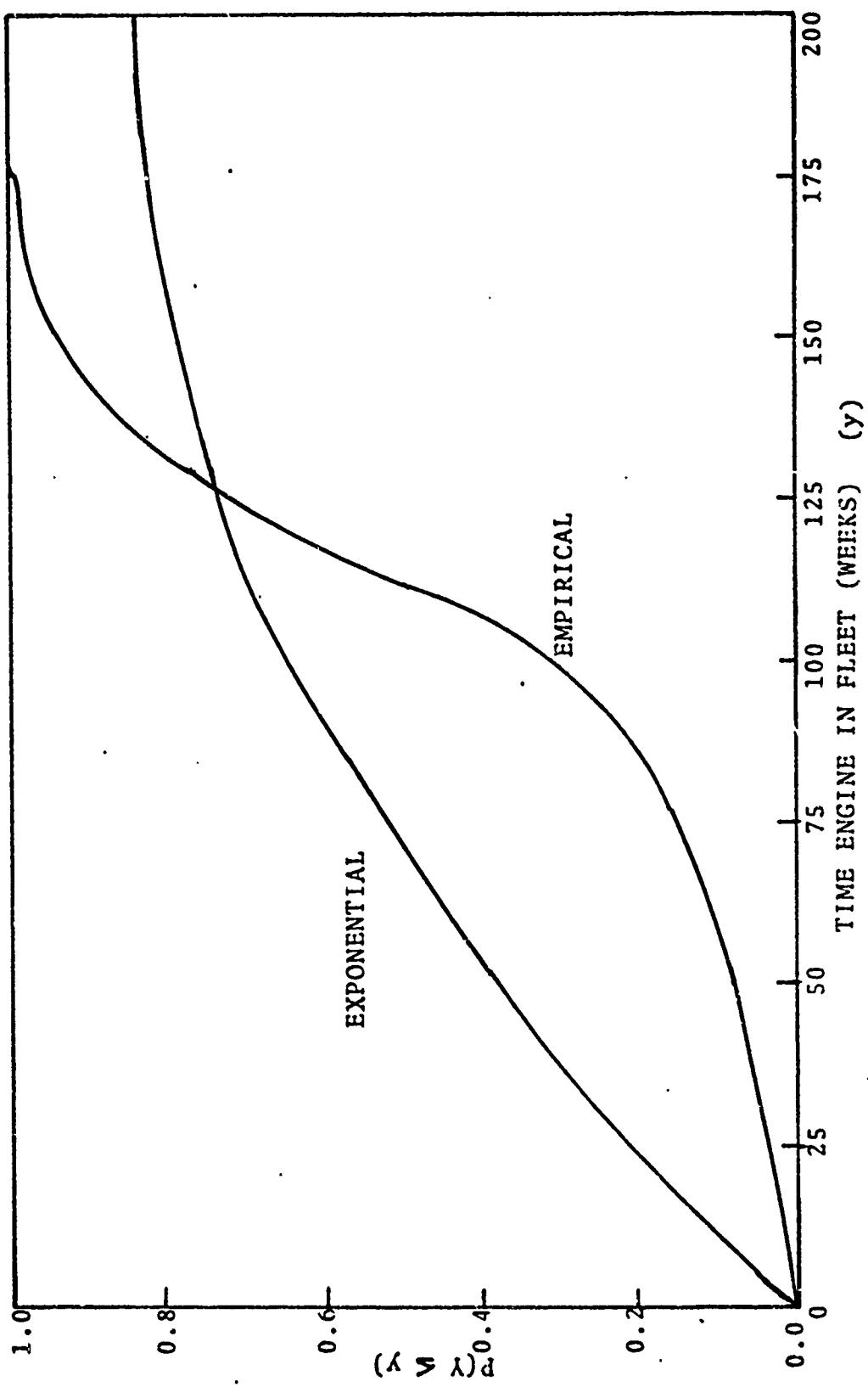


Figure 6.  $P(Y \leq y)$  vs.  $y$ .

for use in the analytical model, we have defined the following:

$$E[X] \text{ (in weeks)} = 52(TBO/TOR) \text{ and}$$

$$E[Y] \text{ (in weeks)} = .9E[X], \text{ for all TBO and TOR combinations.}$$

If the interest is in the number of engines that have to be removed at exactly 30 weeks of operation in the fleet, or some other such specific time, the two models would not necessarily agree. But if the primary interest is in analyzing overall results, e.g., the average number of engines short in the fleet, then the two models give virtually the same results.

Given the current state of the system, no one knows which distribution will best describe the actual operations. But for analyzing the long term behavior of the whole system, either model could be used. The choice of which model to use should be determined by the advantages one might have over the other.

The analytical model is limited in that it provides only steady-state results. For this reason, the simulation is used to analyze the transient time problems of incremental ship delivery, incremental pool purchases, and the additional problem of transportation delays. The analytical model provides a rapid efficient method of measuring the effects of changing various policy variables. The results obtained on the computer, using the analytical model,

require about 2 seconds computer time for each set of parameters investigated, whereas the simulation requires about 2.5 minutes for the same analysis.

#### IV. INTERACTIVE COMPUTER PROGRAM

To provide the policy planner with a real-time system for determining the effects of various parameter changes on the LM-2500 use/repair cycle, an interactive computer program has been developed which uses the analytical model.

To operate the interactive computer system, the policy planner would require the use of a remote terminal which can be connected to the computer in which the program is stored. He would then go through a standard procedure to begin the execution of the program. This procedure is a function of the computer center so it will not be explained here.

Once execution begins, the program will print instructions for entering the use/repair cycle parameters. Seven parameters are to be entered. They are:

1. Number of ships
2. Number of engines per ship
3. Number of spare engines
4. Number of repair lines in the rework facility
5. Average rework time per engine in weeks
6. Time between overhauls in hours
7. Engine operating hours per year.

When all of these parameters have been entered, the program will type out the parameters and their value so that these can be checked for possible errors. If they are in fact correct, the user types a 1 (one), and the program will

calculate the use/repair cycle system performance measures. If they are not correct the user can type a 0 (zero), and the program will return to its start so the correct parameters can be entered. In Appendix A, on page 50, an actual example of this input procedure is included.

On page 52, a sample of the output is also listed. This whole process of input and output requires about five minutes. The decision maker can, if he chooses, restart the program with a new set of parameters. Actual computer time per set of parameters is less than two seconds.

## V. ANALYSIS

### A. STEADY STATE RESULTS

Of the many measures of system performance, the following measures were chosen as providing the most information to the policy planner:

1. Average number of engines short in the fleet ( $\bar{s}$ ),
2. Average number of RFI engines ( $\bar{w}_2$ ),
3. Average percent of rework capacity used ( $U_m$ ),
4. Average number of engines awaiting rework ( $\bar{w}_1$ ).

When looking at these measures for various combinations of system parameters, it must be kept in mind that they are steady state results. That is, this is how the system will look after it has been operating over a long period of time. The effects of the start-up of the system have little or no effect on the value of the steady state system performance measures.

In figures 7 through 10, the four measures above are plotted versus the ratio of TBO to TOR. All four graphs are based on 19 spare engines. The effects of a change in the number of spare engines are investigated below. Table 1 converts a given TBO and TOR into the ratio shown on the graphs. In each figure, a set of nine curves is plotted. Each curve corresponds to a given number of repair lines ( $M$ ) and an average repair time ( $T_1$ ). Each line is labeled  $(M, T_1)$ .

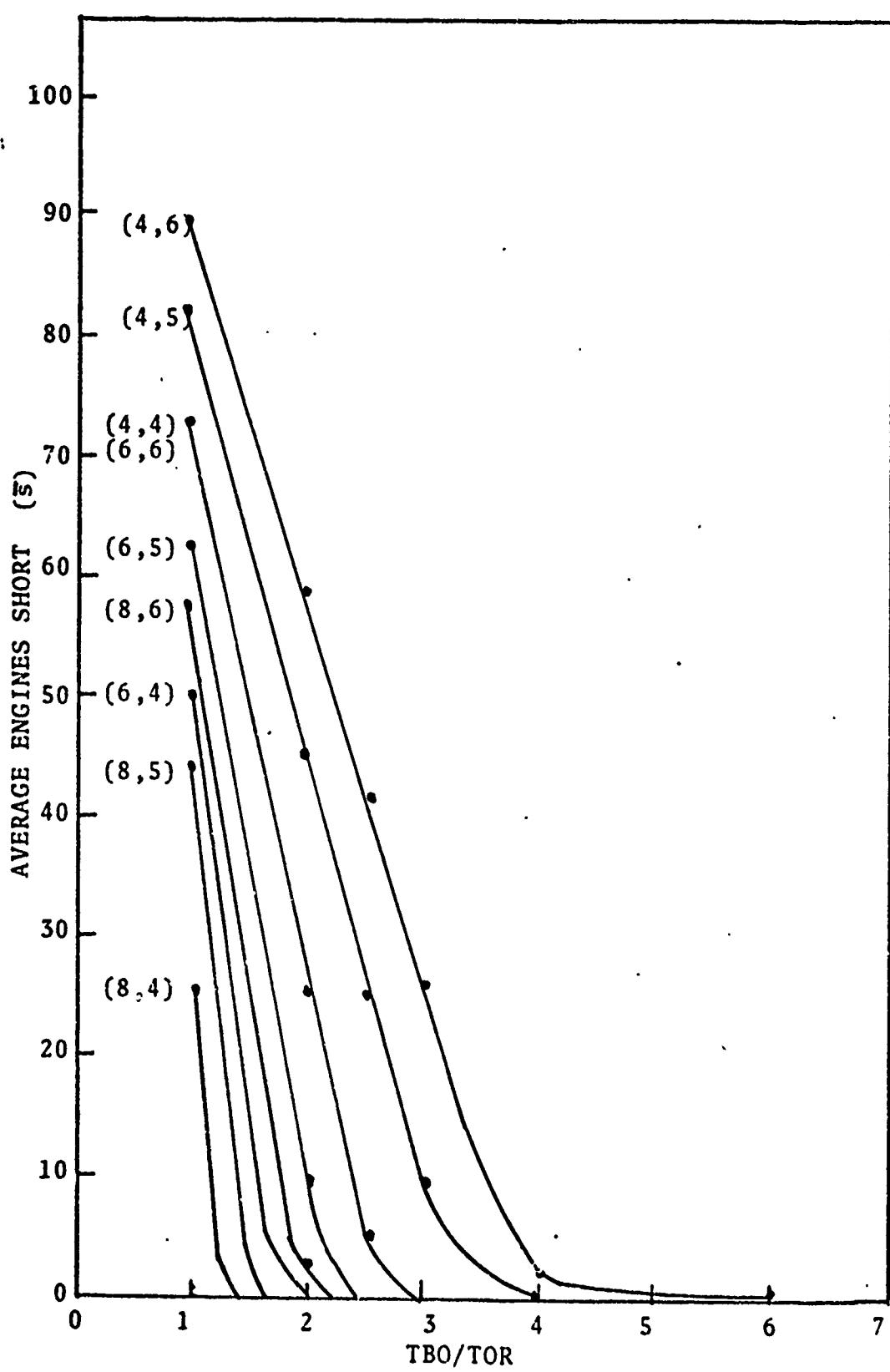


Figure 7.  $\bar{s}$  vs. TBO/TOR.

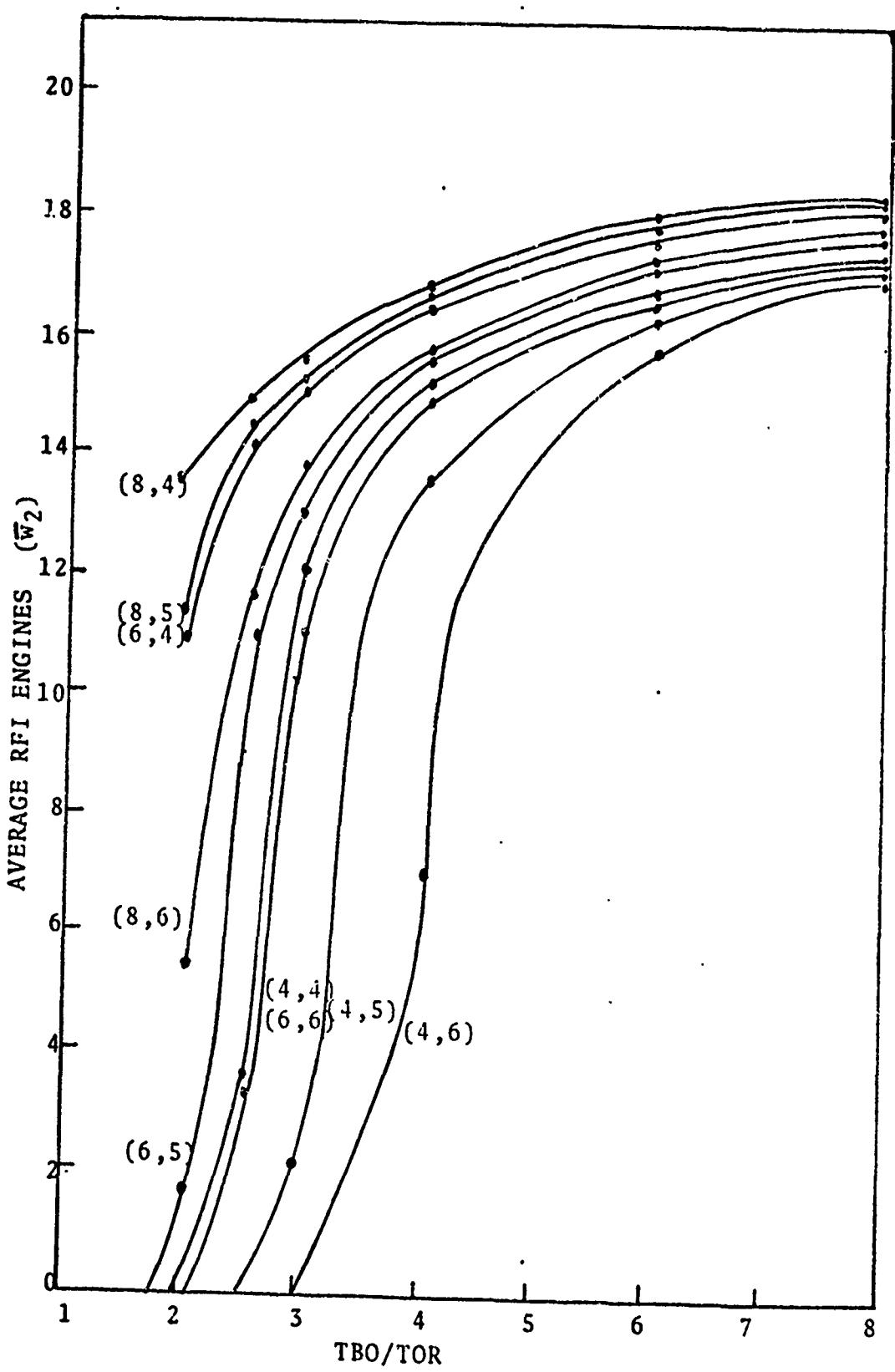


Figure 8.  $\bar{w}_2$  vs. TBO/TOR.

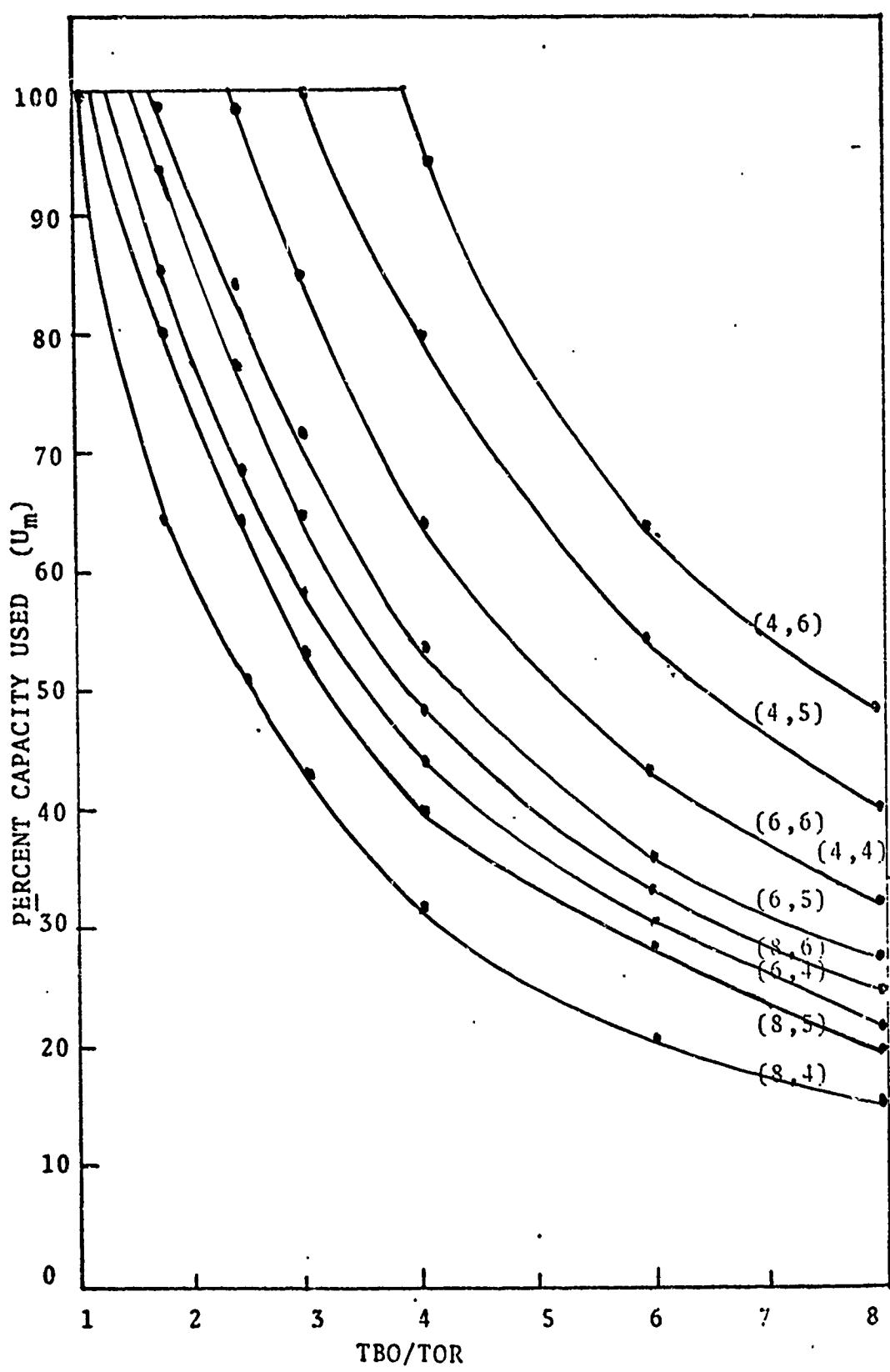


Figure 9.  $U_m$  vs.  $TBO/TOR$ .

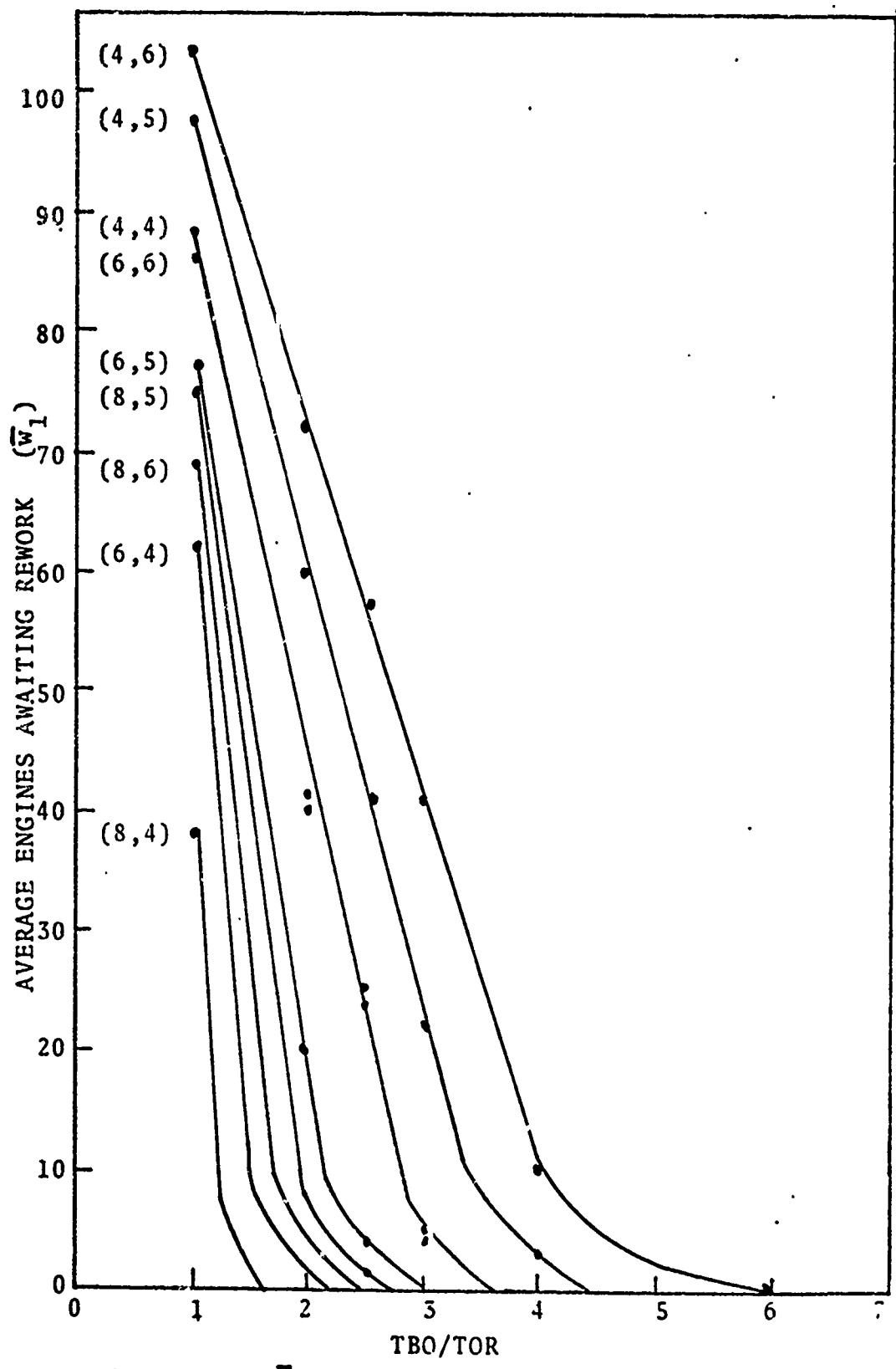


Figure 10.  $\bar{w}_1$  vs.  $TBO/TOR$ .

TABLE 1

Converts a given TBO and TOR to the ratio used in figures 7 through 10

<u>TBO</u>	<u>TOR</u>		
	1650	2000	2500
4000	2.4	2.0	1.6
5000	3.0	2.5	2.0
6000	3.6	3.0	2.4
7000	4.2	3.5	2.8
8000	4.8	4.0	3.2
9000	5.5	4.5	3.6
10000	6.1	5.0	4.0
11000	6.7	5.5	4.4
12000	7.3	6.0	4.8

Figure 7 shows the relation between average number of engines short in the fleet ( $\bar{s}$ ) and TBO/TOR. For each of the nine curves there is a dramatic decrease in  $\bar{s}$  as TBO/TOR increases to about 4, usually an increase in engine reliability. Also it can be seen that there is a strong effect due to the combination of  $M$  and  $T_1$ . That is, when there is adequate repair capacity in terms of size and speed, then the number of engines short in the fleet is reduced rapidly.

The plots of the other three measures in figures 8, 9, and 10 show the same dramatic changes as TBO/TOR is increased to about 4. The combination of  $M$  and  $T_1$  also produce the same sharp effects as the rework facility is made more capable of reworking engines.

For example, let us look at a system where

$$M = 6,$$

$$T_1 = 5,$$

$$TBO = 4000,$$

$$TOR = 2000.$$

We will then change TBO to 6000 and look at the result of this change in TBO/TOR from 2 to 3. Now enter each figure with these two ratios. In figure 7, entering at  $TBO/TOR = 2$  we find the value of  $\bar{s}$  to be 10 on the curve labeled (6,5). When entering the graph at  $TBO/TOR = 3$ , we find that the (6,5) curve has gone to 0 for a ratio less than 3. So the change in TBO has resulted in a reduction of the number of engines short in the fleet by 10. Entering

figure 8 in the same manner, shows that the number of RFI engines is increased from 2 to about 13. From figure 9, we see that the average facility capacity used decreased from 99% to 73%. From figure 10, we see that the average number of engines awaiting rework decreased from 20 to less than 1. It is therefore quite evident that any program to increase the reliability of the engines will be of great benefit to the operating fleet and will cut down on the required rework capacity.

The above graphs were produced using 19 spare engines, with a fleet of 120 engines. A natural question is what effect will a change in the number of spare engines have on the four system measures? For this reason, the four measures are plotted versus the number of spare engines in figures 11 through 14. Each figure uses a value of  $T_2 = 106$  weeks,  $M = 6$ , and  $T_1 = 5$  weeks. In some figures, other values of  $M$  and  $T_1$  are used and will be explained as they are needed.

Figure 11 shows the average percent of facility capacity used ( $U_m$ ) versus spare engines. This measure is insensitive to changes in the number of spare engines. So when comparing  $U_m$ , the number of spare engines need not be considered.

In figure 12, the average number of engines short in the fleet is plotted. This measure does show some effects due to the number of spare engines. However, the effect is larger between 8 and 15 spare engines than between 15

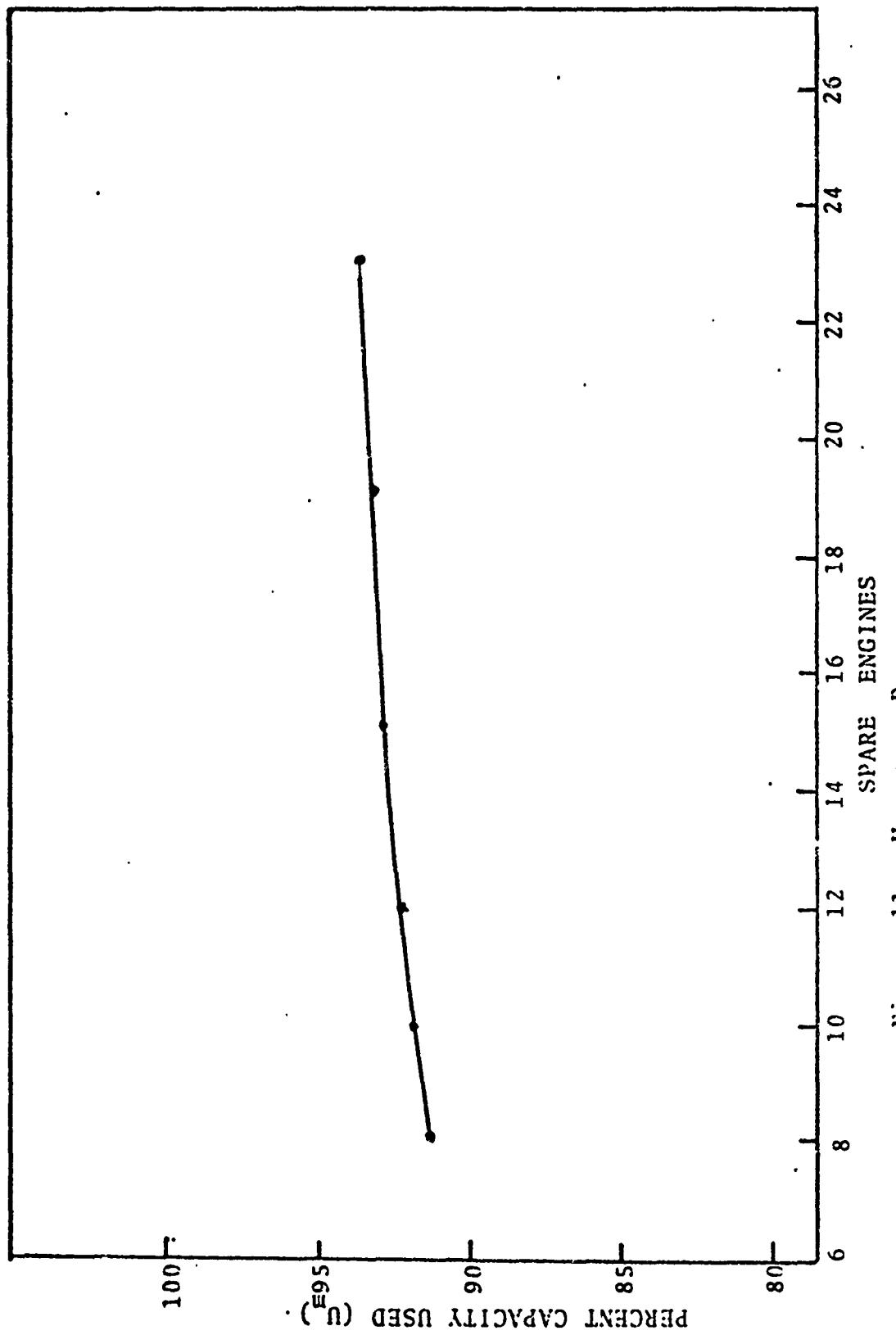


Figure 11.  $U_m$  vs. P.

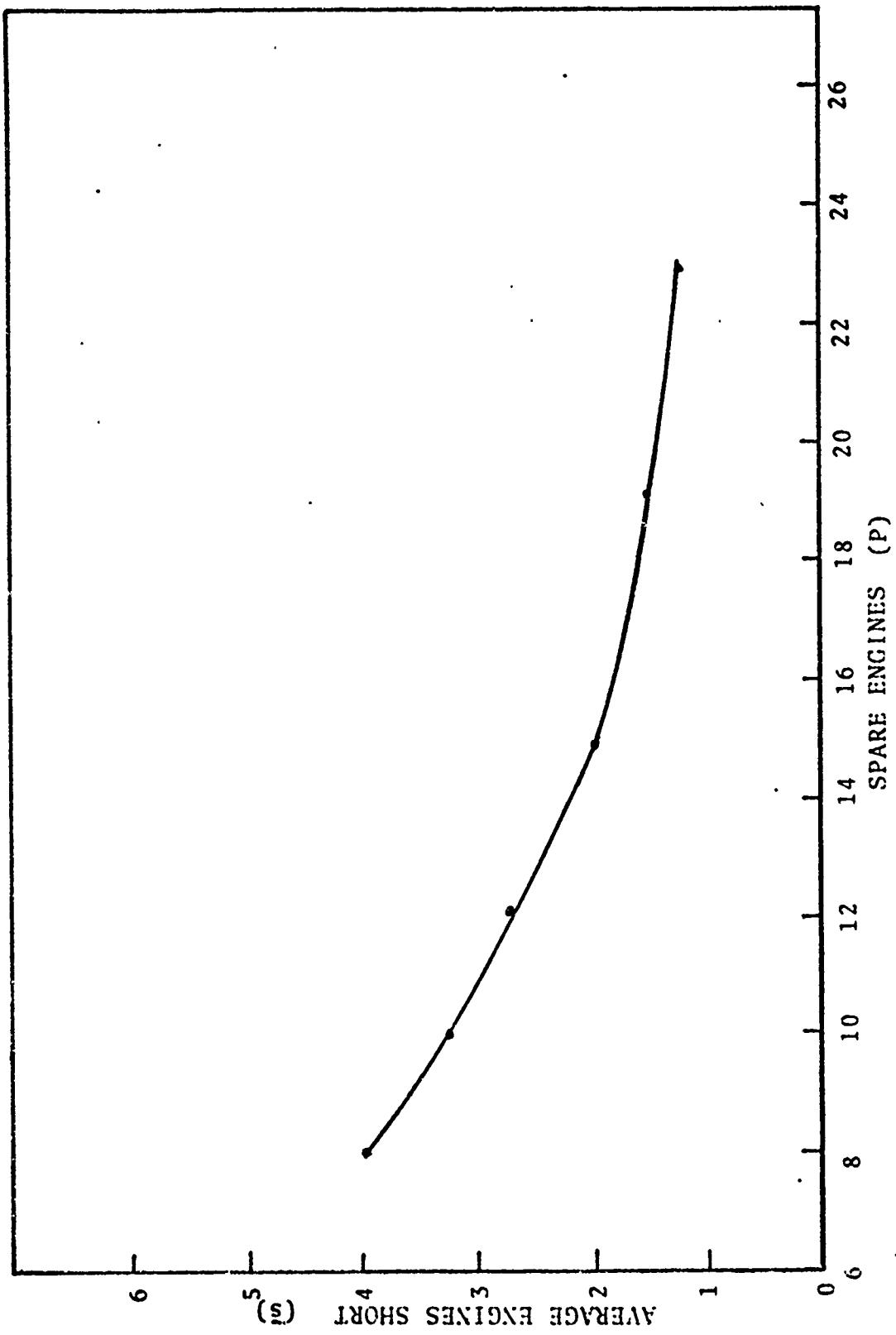


Figure 12.  $\bar{S}$  vs.  $P$ .

and 23 spare engines. When changing the number of spares from 8 to 15, the number short is reduced by only 2. In the range from 15 to 23, this reduction is only one engine. Therefore, an increase in the number of spare engines has very little effect in reducing shortages in the fleet. Figure 7 shows that there is a better return received by developing the right rework facility than buying spare engines.

Figure 13 shows a different sort of result for the average number of RFI engines. Whereas the average number of engines short did not change much, this measure does. In this case, three curves are plotted, the standard (6,5) case, the best case (8,4), and the worst case (4,6). The number of RFI engines is low and unchanging for the (4,6) case. When the rework facility capacity is increased (increase  $M$ , decrease  $T_1$ ), the number of RFI engines begins to increase with the number of spare engines. However it is the parameters of the rework facility that are causing the effects, not the number of spare engines. If the rework facility is large enough and the rework time short enough then the spare engines end up in the RFI pool.

Figure 14 shows that the waiting line at the facility does increase with an increase in the number of spare engines. This is perhaps the best evidence that if there is not enough rework capacity, an increase in the number of spare engines will not help the operating fleet. The engines merely add to the length of the rework waiting

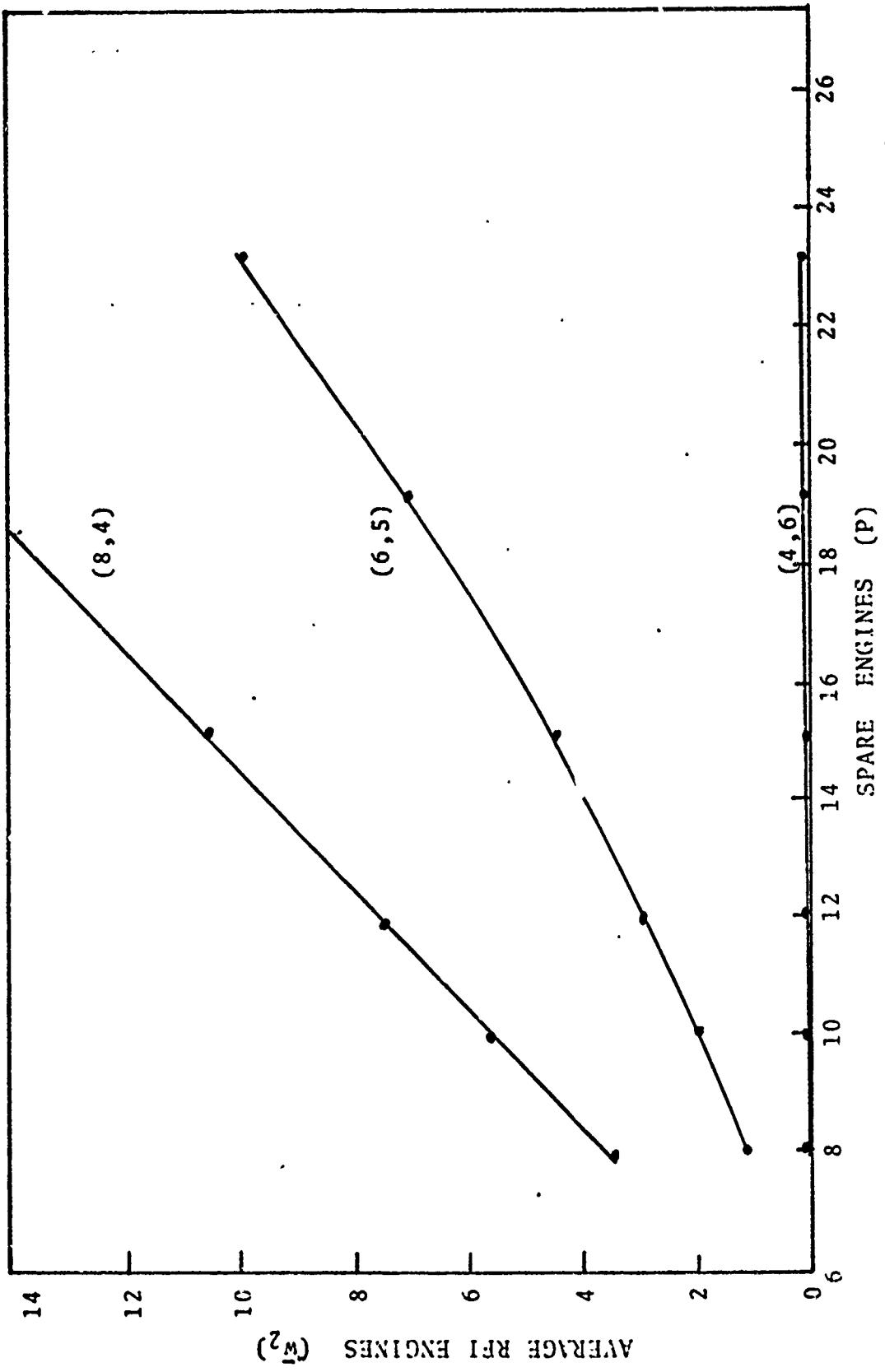


Figure 13.  $\bar{w}_2$  vs. P.

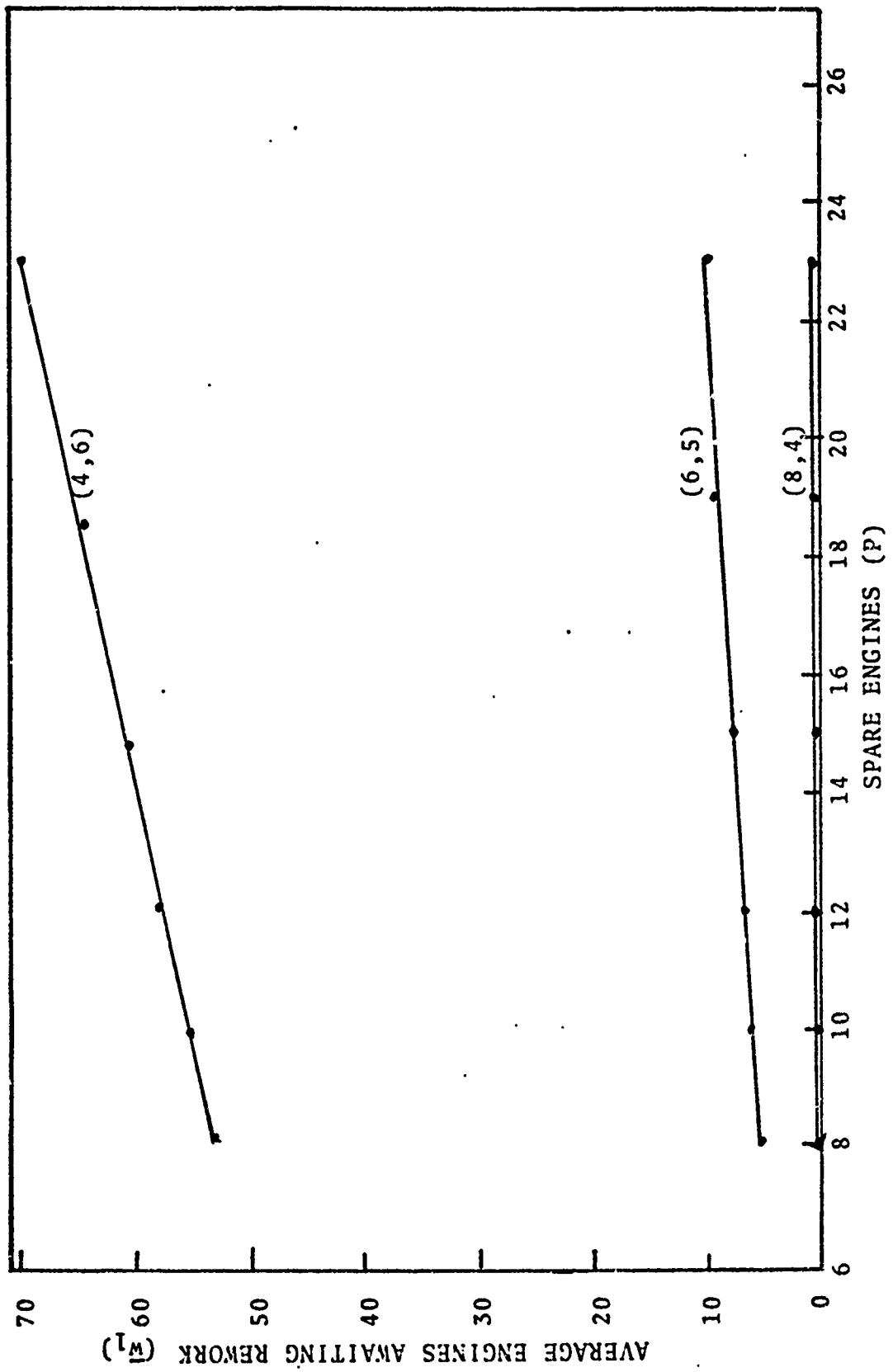


Figure 14.  $\bar{w}_1$  vs. P.

line. The slope of the curves approaches 1 as the rework facility parameters approach their worst case. So in the long run, to buy spares to keep the fleet operating will not accomplish its purpose unless it is accompanied by a corresponding increase in rework facility capacities.

#### B. TRANSPORTATION DELAYS

The analytical model could not include the delays which are incurred between removal from the ship and arrival at the repair facility, and the similar delay incurred between the RFI pool and the fleet when an unscheduled removal is experienced. For these reasons, the simulation model was used to determine the effects of transportation delays.

The simulation model was run with the 120 engine operation, a pool size of 19, repair time of 5 weeks, 6 repair lines, and transportation delays in both of the above cases of 0,2,3,4, and 8 weeks. The average number of engines short in the fleet was observed in each case with the following results:

<u>Transportation Delay</u>	<u>Ave. # engines short in fleet</u>
0	0.3
2	1.7
3	2.8
4	3.8
8	8.3

This indicates that each week of transportation delay costs the operating fleet an average of one engine. This might be interpreted to mean that an increase in pool size of one for each week of transportation delay would counteract

the effect of 1 week transportation delay. To test this, for the transportation delay of eight weeks, the pool size was increased from 19 to 27 and the simulation repeated. The average number of engines short decreased to 4.3 from 8.3. Thus, two additional engines in the pool were required to reduce the number short in the fleet by 1.

### C. TRANSIENT RESULTS

Two different spare-engine buy plans were tested with the simulation model. Each plan was for a total pool size of 19. The plans were tested with the 4,000 hour TBO, 2,000 annual operating hours per engine TOR, repair time of 5 weeks, 6 repair lines, transportation delay of two weeks, and the DD-963 planned delivery schedule. The buy plans tested were:

	Delivery Dates		Number of engines	
	Test 1	Test 2	Option 1	Option 2
1 Jul 75	1 Jan 76		5	2
" 76	" 77		5	3
" 77	" 78		3	3
" 78	" 79		3	5
" 79	" 80		3	6

Using the percentage of time there are no engines in the pool over a twenty year period as a measure to compare the effects of the two buying plans, the following results were obtained:

Option 1, Test 1	92.7%
Option 2, Test 1	91.9%
Option 1, Test 2	92.8%
Option 2, Test 2	92.0%

This indicates that the system is relatively insensitive to the differences in these two buy plans over reasonably long time periods. The particular plan selected should probably be determined by economic considerations in the short term, say the first 5 years. It was felt that the simulation model did not provide reliable analysis of the short term effects of the two buy plans.

## VI. SUMMARY

Using as measures a) the average number of engines short in the fleet, b) the average number of engines in the RFI pool, c) the percentage of repair capacity used, and d) the average number of engines awaiting rework, the significant findings of this study can be summarized as follows:

1. In the long run, the four measures are extremely sensitive to the ratio of time between overhaul to annual engine operating hours. The system is improved by a higher ratio.
2. In the long run, the four measures are extremely sensitive to the repair facility capacity as measured by the number of repair lines and average engine repair time.
3. In the long run, the percentage of repair capacity used and the average number of engines short in the fleet are relatively insensitive to the number of spare engines purchased. However, the average number of engines in the RFI pool and the average number of engines awaiting rework are extremely sensitive to the number of spare engines purchased. The additional spares end up in an inactive status over the long run. If there is excess repair capacity, they end up in the RFI pool. If there is insufficient repair capacity, they end up in the repair facility waiting line. For cases in between they are split between the RFI pool and the repair facility waiting line.

4. The interactive computer system developed for use with the analytical model provides a useful method for the policy planner to immediately see the effects of changes in system parameters.

EXECUTION BEGINS...

DATE 08/30/72

DETERMINATION OF OPERATIONAL CHARACTERISTICS OF LM 2500  
USE/REWORK CYCLE.

AFTER EACH PARAMETER EXAMPLE, ENTER YOUR PARAMETER CHOICE  
IN THE FORMAT SHOWN.

1. NUMBER OF SHIPS  
EXAMPLE 30 (YOU ENTER NUMBER XX AND HIT RETURN)  
30

2. NUMBER OF ENGINES PER SHIP  
EXAMPLE 4 (X)  
4

3. NUMBER OF SPARE ENGINES  
EXAMPLE 20 (XX)  
19

4. NUMBER OF REPAIR LINES  
EXAMPLE 06 (XX)  
06

5. AVERAGE REWORK TIME PER ENGINE (WEEKS)  
EXAMPLE 5 (X)  
5

6. TBO (TIME BETWEEN OVERHAULS, HOURS)  
EXAMPLE 4000 (XXXX)  
4000

7. ENGINE OPERATING HOURS PER YEAR  
EXAMPLE 2000 (XXXX)  
2000

CHECK YOUR PARAMETER CHOICES

NUMBER OF SHIPS	30
NUMBER OF ENGINES/SHIP	4
NUMBER OF SPARES	19
NUMBER OF REPAIR LINES	6
AVERAGE REWORK TIME	5
TBO	4000
ENGINE OPERATING HOURS	2000

ARE THESE CORRECT? IF YES TYPE 1  
IF NO TYPE 0, PROGRAM WILL RESTART  
1

LM-250C USE/REWORK SYSTEM PERFORMANCE MEASURES

REWORK FACILITY STATISTICS

AVERAGE NUMBER OF ENGINES AWAITING REWORK	- 20.0
AVERAGE NUMBER OF ENGINES BEING REWORKED	- 5.9
AVERAGE % CAPACITY USED	- 99.1
% TIME ALL REPAIR LINES BUSY	- 97.3

FLEET/POOL STATISTICS

AVERAGE NUMBER OF ENGINES IN FLEET	- 111.3
AVERAGE NUMBER OF ENGINES OUT IN FLEET	- 8.7
AVERAGE NUMBER OF ENGINES READY FOR ISSUE	- 1.8
AVERAGE % ENGINES OPERATIONAL	- 80.0
AVERAGE CYCLE TIME (WEEKS)	- 116.9
% TIME FLEET IS SHORT	- 1
OR MORE ENGINES	- 71.9
6	- 57.6
11	- 40.0
16	- 22.6

DO YOU DESIRE TO RUN ANOTHER SET OF PARAMETERS?  
IF YES, TYPE 1; IF NO, TYPE 0  
0

APPENDIX B  
COMPUTER PROGRAMS

```

***** GPS SIMULATION MODEL COMPUTER PROGRAM ****
***** SIMULATE *****

* INITIALIZE TRANSPORTATION DELAY
* INITIALIZE RAND NUMBER POOL
* INITIALIZE LOADING SCHEDULE FOR ROTATABLE POOL
* INITIALIZE LOADS 1, 0/XH$LOAD2, 5/XH$LOAD3, 5/XH$LOAD4, 3
* INITIALIZE LOADS 5, 3/XH$LOAD6, 3/XH$LOAD7, 3/XH$LOAD8, 3
* INITIALIZE TIME 2, 30/XH$TIME3, 52/XH$TIME4, 52/XH$TIME5, 52
* INITIALIZE TIME 6, 52
* INITIALIZE STORAGE 6
* FACILITY STORAGE FUNCTION 0
* NAVY SHIPS FUNCTION 0
* 1.7 8 24 3 8 4 4 10 8 12 6 16
* 13 4 14 4 15 4 16 4 11 4 18 4
* 19 8 120 120 120 120 120 120 120 120 120 120 120
* 25 4 26 4 27 4 28 4 29 4 30 4
* 1.009, 0.066, /0.028, 0.072, /0.047, 0.078, /0.074, 0.084, /0.120, 0.090, /0.175, 0.096, /
* 2.49, 1.02, /3.42, 1.08, /4.62, 1.14, /5.82, 1.20, /7.02, 1.26, /8.04, 1.32, /
* 8.78, 1.38, /9.15, 1.44, /9.44, 1.50, /9.63, 1.56, /9.82, 1.62, /9.91, 1.68, /
* 1.000, 1.74 GENERATE FNSHIPS1, ,30 INT+, K1,H
* FIRST MARK TEST E NSFIRST, K1, HOP
* SPLIT E 1, INFO 1, HOLD SEND SIGNAL TO START TIMING
* SPLIT 1, HOLD DIVERT ONE TRANS. TO LOAD STOCK
* HOP MARK SPLIT 3, NEXT CREATE FOUR ENGINE PER SHIP
* NEXT MARK COUNT FLEET ENTER SAVEVALUE OPER, FN1, H
* COUNT FLEET ENTER SAVEVALUE TRANSFER *XH$RND, RNAV, RFAIL TIME TO FAIL FITBO, OPSCHED
* RNAV ADVANCE JOIN LEAVE LOGIC S ADVANCE S QUEUE FAC ENTER FAC

```

	FAC	REPAIR TIME
DEPART	XH\$REWICK	ENGINES NOW RFI
ADVANCE	FAC	REPLACE FAILED ENGS.
LEARN	POOL	
STOCK	1	
QUEUE LS	POOL	
GATE LS	V9,K1,OPEN	
DEPART	1	
TEST C		
R		
LOG C		
OPEN		
MARK		
ADVANCE	XH\$TRANS	
TRANSFER	,FLEET	
MARK	V\$MEAN, V\$MEAN	
ADVANCE	,JOIN	
TRANSFER		
MARK		
HOLD		
SPLIT	X1\$LOAD1, STOC1	
ADVANCE	XH\$TIME2	
SPLIT	XH\$LOAD2, STOC1	
ADVANCE	XH\$TIME3	
SPLIT	XH\$LOAD3, STOC1	
ADVANCE	XH\$TIME4	
SPLIT	XH\$LOAD4, STOC1	
ADVANCE	XH\$TIME5	
SPLIT	XH\$LOAD5, STOC1	
ADVANCE	XH\$TIME6	
SPLIT	XH\$LOAD6, STOC1	
TERMINATE	O	
STOC1		
MARK		
TRANSFER	,STOCK	
INFO		
MARK		
ASSIGN	7,1040	
SAVEVALUE	I+,K1	
XXX1		
ADVANCE	1	
TEST L	\$\$NAV, NSCOUNT, OUT	
OUT	7,XXX1	
LOOP	,FINAL	
TRANSFER		
FINAL		
MARK		
SAVEVALUE	INT,K1,H	
PRINT	1,2,Q	
TERMINATE	1,2,S	
VARIABLE	1,NSCOUNT-\$\$NAV	
MEAN 1	XH\$OPER/2	
START	1, NP	
END		

```

*** * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*   FORTRAN ANALYTICAL MODEL COMPUTER PROGRAM
*   COMPUTER PROGRAM FOR THE CALCULATION OF THE SYSTEM PERFORMANCE
C MEASURES FOR THE LM-2500 USE/REWORK CYCLE
C THE MODEL USES THE METHOD DEVELOPED BY KOENIGSBERG IN 1960.
C & 9 ADDITION THE PROGRAM IS DESIGNED FOR USE AS AN INTERACTIVE
C TIME SHARE SYSTEM.
C
      IMPLICIT REAL*8 (A,B,C,D,S,U,H)
      INTEGER Z(6)
      DIMENSION A(1000),SP(1000)
      CALL ERSET(208,256,-1,1,1,0)
      1111 CONTINUE
C
C READ THE INPUTS
C
      CALL IXCLK(2)
      WRITE(6,9100)Z(1),Z(2)
9100  FORMAT(10.0,1A4,"DETERMINATION OF SYSTEM PERFORMANCE
      1 MEASURES OF LM 2500 USE/REWORK CYCLE // AFTER EACH PARAMETER E
      2 XAMPLE ENTER YOUR PARAMETER CHOICE IN THE FORMAT SHOWN.")
      WRITE(6,9101)
      9101 FORMAT(10.1,"NUMBER OF SHIPS",/, "EXAMPLE 30 (YOU ENTER NUMBER
      1 XXX AND HIT RETURN),")
      1 READ(5,9102)KS
      9102 FORMAT(1I2)
      WRITE(6,9103) NUMBER OF ENGINES PER SHIP,/, "EXAMPLE 4 (X)" )
      9103 FORMAT(10.0)
      READ(5,9104)KE
      9104 FORMAT(1I1)
      WRITE(6,9105)
      9105 FORMAT(1I1)
      READ(5,9102)KP
      WRITE(6,9106)
      9106 FORMAT(10.0)
      READ(5,9107)N
      WRITE(6,9108)
      9108 FORMAT(10.0)
      READ(5,9102)M
      WRITE(6,9109)
      9109 FORMAT(10.0)
      1E5(X),1
      9210 FORMAT(1I1)
      WRITE(6,9106)
      9106 FORMAT(10.0)
      1000 (XXX),/

```



```

A(I)=A(IM) * ARHO / M
I=1+1
DO 10 2000
C NAK= I <= N
C
C 3000 IF(I>L) GO TO 4000
IM=I-
A(I)=A(IM) * RHO * (N+2-I) / M
I=I+1
GO TO 3000
C
C 4030 DENOM= 0.0
DO 4500 I=L+1
DENOM= DENOM + A(I)
CONTINUE
DO 4600 I=L
SP(I)=A(I) / DENOM
CONTINUE
STOT= 0.0
DO 5000 I=1,L
STOT= STOT + SP(I)
CONTINUE
ACC=DABS(1.0-STOT)
IF(ACC.GT.0.001)GO TO 5500
DO 5700
      WRITE(6,5600)STOT
5600 FORMAT(10.1560)STOT
      POSSIBLE LOSS OF ACCURACY, /, THE SUM OF
      STATE PROBABILITIES IS, D25.15, /)
5700 CONTINUE
C CALCULATE THE SYSTEM PERFORMANCE MEASURES.
C
C BARN1=0.0
DO 6000 I=1,N
IP=I+
BARN1=BARN1 + (I * SP(IP))
CONTINUE
C BARN2= N - BARN1
C BARWI= 0.0
MM=M+1
DO 6100 I=MM,N
IP=I+
BARWI= BARWI + ((I-M) * SP(IP))
6100 CONTINUE

```

```

C      BARW2= NA * SP(1)
DO 6200 I=1,NA
IP= I + 1
BARW2= BARW2 + ((NA-I) *SP(IP))
C      6200 CONTINUE
C      H1= BARN1 - BARW1
H2= BARN2 - BARW2
H2S= KA - H2
C      D1=0.0
DO 6300 I=M,N
IP= I + 1
D1= D1 + SP(IP)
D1= D1 / SP(IP)
6300 CONTINUE
D1= D1 * 100.0
C      D2= SP(1)
DO 6400 I=1,NA
IP= I + 1
D2= D2 + SP(IP)
6400 CONTINUE
C      UM= H1 / M
UM= UM * 100.0
UN= H2 / N
UN= UN * 100.0
C      DLA= FAILR * KA * D2
NAPP= NA + 2
DO 7000 I=NAPP,N
DLA= DLA + (FAILR * SP(I) * (L-I))
7000 CONTINUE
DCYCL= N / DLA
C      PRINT THE OUTPUT
C      WRITE(6,9800)
9800 FORMAT(10,/,LM 2500 USE/REWORK SYSTEM PERFORMANCE MEASURES:,/
1/) WRITE(6,9801) BARN1,H1,UM'DI
9801 FORMAT(0,'REWORK FACILITY STATISTICS'//';' AVERAGE NUMBER OF ENG
INES AWAITING REWORK' F9.1,'; AVERAGE NUMBER OF ENGINES BEING RE
WORKED' F10.1,'; AVERAGE % CAPACITY USED' ,F27.1,'; % TIME ALL REP
3AIR LINES BUSY' ,F22.1)
WRITE(6,9802) H2,H2S,BARW2,UN,DCYCL

```

```

9802 FORMAT(10.2, F16.1, //, *AVERAGE NUMBER OF ENGINES
1 IN FLEET, F16.1, /*AVERAGE NUMBER OF ENGINES SHORT IN FLEET, F10.
2 /*AVERAGE NUMBER OF ENGINES READY FOR ISSUE, F21.1, /*AVERAGE
3 /*ENGINES OPERATIONAL, F21.1, /*AVERAGE CYCLE TIME, F24.1
4)

IS=1
DMIS=(1.0-D2)*100.0
WRITE(6,9803)IS,DMIS
FORMAT(10.0, %TIME FLEET IS SHORT, 14, * OR MORE ENGINES, F8.1)
9803
IF(DMIS<LT,30.0)GO TO 7400
SPS=0.0
DO 750 J I=1,5
NAA=IS+1+NA
SPS=SPS+SP(NAA)
CONTINUE
SPSP=SPS * 100.0
IS=IS+5
DMIS=DMIS-SPSP
WRITE(6,9804)IS,DMIS
9804 FORMAT(10.0,125,F25.1)
GO TO 7300
CONTINUE
7400
WRITE(6,9805)
FORMAT(10.0, *DO YOU DESIRE TO RUN ANOTHER SET OF PARAMETERS?, /, *)
98051 IF YES, TYPE 1; IF NO, TYPE 0, )
READ(5,9104)KT
IF(KT.EQ.1) GO TO 1111
STOP
END

```

### LIST OF REFERENCES

1. Cushing, J.S., Gautier, W. K., and Long, D. A., A Simulation Study of the LM-2500 Gas Turbine Engine Inventory System, M.S. Thesis, Naval Postgraduate School, Monterey, California, March 1972.
2. Koenigsberg, E., "Finite Queues and Cyclic Queues," Operations Research, Vol. 8, No. 2, pp. 246-259, 1960.